There are many sets which are NOT computable

What is NOT computable? The following set called ALAN (by Cohen, D. (1997)[Introduction to Computer Theory (2nd ed.), New York: John Wiley]) is NOT computable because there is no Turing Machine (TM) for the set.

ALAN = { TM code-words that are NOT accepted by their corresponding TMs }

ALAN = { <M_i> I <M_i> is NOT accepted by the M_i }

<M_i> is the code of M_i

Proof Idea: Assume that ALAN is computable; it means that there is a TM, T, that accepts ALAN. Every TM can be coded; so, T can also be coded. Assume that <T> is the code for T . The question we ask is: " Is <T> in AL AN? " There are two possibilities:

YES

NO

CASE 1: <T> is in ALAN (i.e. code(T) is in ALAN)

CLAIM	REASON
1. T accepts ALAN	1. Definition of T
2. <T> is in ALAN	2. Hypothesis: CASE 1
3. T accepts <T>	3. From 1 and 2
4. T does NOT accept <T> (From the definition of ALAN) <T> is NOT accepted by T	4. Definition of ALAN
5. Contradiction	5. From 3 & 4

CASE 2: <T> is NOT in ALAN

CLAIM	REASON
1. T accepts ALAN	1. Definition of T
2. <T> is NOT in ALAN	2. Hypothesis: CASE 2
3. T does NOT accept <T>	3. From 1 & 2 *(T accepts only the strings of ALAN (no other strings) because T was specifically built for ALAN. T does NOT accept <T>, because <T> is NOT in ALAN.)*
4. <T> is in ALAN	4. From 3 *(If T does NOT accept <T>, then <T> is in ALAN, according the definition of ALAN)*
5. <T> is NOT in ALAN	5. Hypothesis: CASE 2
6. Contradiction	5. From 4 & 5

ALAN is not computable because such an assumption leads to a contradiction.
No TM exists for ALAN.