

There are many sets which are NOT computable

What is NOT computable? The following set called ALAN (by Cohen, D. (1997)[Introduction to Computer Theory (2nd ed.), New York: John Wiley]) is NOT computable because there is no Turing Machine (TM) for the set.

ALAN = { TM code-words that are NOT accepted by their corresponding TMs }

ALAN = { $\langle M_i \rangle$ | $\langle M_i \rangle$ is NOT accepted by the M_i }

$\langle M_i \rangle$ is the code of M_i

Proof Idea: Assume that ALAN is computable; it means that there is a TM, T , that accepts ALAN. Every TM can be coded; so, T can also be coded. Assume that $\langle T \rangle$ is the code for T . The question we ask is: "Is $\langle T \rangle$ in ALAN?" There are two possibilities:

YES

CASE 1: $\langle T \rangle$ is in ALAN (i.e. code(T) is in ALAN)

CLAIM	REASON
1. T accepts ALAN	1. Definition of T
2. $\langle T \rangle$ is in ALAN	2. Hypothesis: CASE 1
3. T accepts $\langle T \rangle$	3. From 1 and 2
4. T does NOT accept $\langle T \rangle$ (From the definition of ALAN) $\langle T \rangle$ is NOT accepted by T	4. Definition of ALAN
5. Contradiction	5. From 3 & 4

NO



CASE 2: $\langle T \rangle$ is NOT in ALAN

CLAIM	REASON
1. T accepts ALAN	1. Definition of T
2. $\langle T \rangle$ is NOT in ALAN	2. Hypothesis: CASE 2
3. T does NOT accept $\langle T \rangle$	3. From 1 & 2 <i>(T accepts only the strings of ALAN (no other strings) because T was specifically built for ALAN. T does NOT accept $\langle T \rangle$, because $\langle T \rangle$ is NOT in ALAN.)</i>
4. $\langle T \rangle$ is in ALAN	4. From 3 <i>(If T does NOT accept $\langle T \rangle$, then $\langle T \rangle$ is in ALAN, according to the definition of ALAN)</i>
5. $\langle T \rangle$ is NOT in ALAN	5. Hypothesis: CASE 2
6. Contradiction	5. From 4 & 5

ALAN is not computable because such an assumption leads to a contradiction. No TM exists for ALAN. The complement of ALAN, ALAN', { TM code-words that are accepted by their corresponding TMs } is proven to be undecidable which also implies that ALAN is not Turing recognizable (Cohen 1997).